CBSE 2026 EXAMINATIONS
XII MATHEMATICS (041)
Series PQSR5 ♦ Q.P. Code 65/5/1; 65/5/2; 65/5/3

Time Allowed : 180 Minutes **Max. Marks : 80**

General Instructions :

1. This Question paper contains **five sections - A, B, C, D and E**. Each section is compulsory. However, there are **internal choices** in some questions.
2. Section A has **18 MCQs** and **02 Assertion-Reason (A-R)** based questions of **1 mark** each.
   Section B has **05 questions** of **2 marks** each.
   Section C has **06 questions** of **3 marks** each.
   Section D has **04 questions** of **5 marks** each.
   Section E has **03 Case-study / Source-based / Passage-based** questions with **sub-parts** (**4 marks** each).
3. There is no overall choice. However, **internal choice** has been provided in
   - **02 Questions of Section B**
   - **03 Questions of Section C**
   - **02 Questions of Section D**
   - **02 Questions of Section E**

   You have to attempt only one of the alternatives in all such questions.

☑ Q.P. Code - 65/5/1

SECTION A

This section has 20 multiple choice questions of 1 mark each.

01. If matrix $A=\begin{bmatrix}-p & q\\ r & p\end{bmatrix}$ is such that $A^2=I$, then

(a) $1+p^2+qr=0$ (b) $1-p^2-qr=0$ (c) $1-p^2+qr=0$ (d) $1+p^2-qr=0$

Sol. (b) Since $A^2=I \Rightarrow AA=I$

$$\Rightarrow\begin{bmatrix}-p & q\\ r & p\end{bmatrix}\begin{bmatrix}-p & q\\ r & p\end{bmatrix}=I$$

$$\Rightarrow\begin{bmatrix}p^2+qr & 0\\ 0 & rq+p^2\end{bmatrix}=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$$

By equality of matrices, we get $p^2+qr=1 \quad \therefore 1-p^2-qr=0$.

02. If A is a square matrix such that $A^2=A$, then $(A-I)^3-A$ is equal to

(a) I (b) $-I$ (c) A (d) A^2

Sol. (b) Since $AI=IA$, so we can use $(a-b)^3=a^3-3a^2b+3ab^2-b^3$ to expand $(A-I)^3$.

Now $(A-I)^3=A^3-3A^2I+3AI^2-I^3$

$\Rightarrow (A-I)^3=A-3AI+3AI-I$ [$\because A^3=A^2A=AA=A^2=A$

$\Rightarrow (A-I)^3=A-3A+3A-I$ [$\because AI=IA=A$

$\Rightarrow (A-I)^3-A=-I$.

03. For the inverse trigonometric functions, which of the following Principal Value Branch is **not** correctly defined?

(a) $\tan^{-1}:\mathbb{R}\to\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$ (b) $\sec^{-1}:\mathbb{R}-(-1,1)\to[0,\pi]-\left\{\frac{\pi}{2}\right\}$

(c) $\cot^{-1}: \mathbb{R} \to (0, \pi)$ (d) $\text{cosec}^{-1}: \mathbb{R} - (-1, 1) \to \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

Sol. (d) Since the correct way to define is $\text{cosec}^{-1}: \mathbb{R} - (-1, 1) \to \left[-\frac{\pi}{2}, \frac{\pi}{2}\right] - \{0\}$.

04. Let $A = \begin{bmatrix} 0 & -3 & 4 \\ 1 & 0 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} -3 & 0 & 1 \\ 2 & 4 & 0 \end{bmatrix}$. If $A + B + C = O$, then matrix C is

(a) $\begin{bmatrix} -3 & -3 & 5 \\ 3 & 4 & 2 \end{bmatrix}$ (b) $\begin{bmatrix} 3 & 3 & 5 \\ -3 & -4 & -2 \end{bmatrix}$ (c) $\begin{bmatrix} 3 & 3 & -5 \\ -3 & -4 & -2 \end{bmatrix}$ (d) $\begin{bmatrix} -3 & -3 & -5 \\ 3 & 4 & 2 \end{bmatrix}$

Sol. (c) $A + B + C = O \quad \Rightarrow C = -A - B = \begin{bmatrix} 0 & 3 & -4 \\ -1 & 0 & -2 \end{bmatrix} - \begin{bmatrix} -3 & 0 & 1 \\ 2 & 4 & 0 \end{bmatrix}$

$\therefore C = \begin{bmatrix} 3 & 3 & -5 \\ -3 & -4 & -2 \end{bmatrix}$.

05. If A is a non-singular matrix, then which of the following is **not** true?

(a) adj. A is singular (b) $(\text{adj.}\, A)^{-1} = (\text{adj.}\, A^{-1})$

(c) $|A| \neq 0$ (d) A^{-1} exists

Sol. (a) The statemet "adj. A is singular" is **not** true.

Note that, $|\text{adj.}\, A| = |A|^{n-1}$, where n is order of A. When $|A| \neq 0$, then $|\text{adj.}\, A| \neq 0$.

That is, the determinant of the adjoint is not zero. Hence, adj. A is also non-singular.

06. If $f(x) = \begin{cases} \frac{x^2 - 4x - 5}{x + 1}, & x \neq -1 \\ k, & x = -1 \end{cases}$ is continuous at $x = -1$, then the value of k is

(a) Any real value (b) 6 (c) –1 (d) –6

Sol. (d) Since f is continuous at $x = -1$. So, $f(-1) = \lim_{x \to -1} f(x)$.

That is, $k = \lim_{x \to -1} \frac{x^2 - 4x - 5}{x + 1}$

$\Rightarrow k = \lim_{x \to -1} \frac{\frac{d}{dx}\left[x^2 - 4x - 5\right]}{\frac{d}{dx}\left[x + 1\right]}$ $\left[\because \text{It is } \frac{0}{0} \text{ form, so we can use L' Hospital rule}\right.$

$\Rightarrow k = \lim_{x \to -1} \frac{2x - 4}{1}$

$\Rightarrow k = 2(-1) - 4$

$\therefore k = -6$.

07. If the area of ΔABC with vertices A(3, 1), B(–2, 1) and C(0, k) is 5 sq. units, then values of k are

(a) 3, 1 (b) –1, 3 (c) –1, 2 (d) 0, 2

Sol. (b) Using area of ΔABC, absolute value of $\frac{1}{2}\begin{vmatrix} 3 & 1 & 1 \\ -2 & 1 & 1 \\ 0 & k & 1 \end{vmatrix} = 5$

$\Rightarrow 0 - k(3 + 2) + 1(3 + 2) = \pm 10$

$\Rightarrow -5k + 5 = \pm 10$

$\Rightarrow k - 1 = \mp 2$

$\therefore k = -1, 3$.

08. Derivative of $\cos^{-1}\left(\frac{\sin x + \cos x}{\sqrt{2}}\right), -\frac{\pi}{4} < x < \frac{\pi}{4}$ with respect to x is

(a) –1 (b) 1 (c) $\frac{\pi}{4}$ (d) $-\frac{\pi}{4}$

Sol. (a) Let $y = \cos^{-1}\left(\frac{\sin x + \cos x}{\sqrt{2}}\right)$

$\Rightarrow y = \cos^{-1}\left(\frac{1}{\sqrt{2}}\sin x + \frac{1}{\sqrt{2}}\cos x\right) = \cos^{-1}\left(\sin\frac{\pi}{4}\sin x + \cos\frac{\pi}{4}\cos x\right) = \cos^{-1}\cos\left(\frac{\pi}{4} - x\right)$

$\Rightarrow y = \left(\frac{\pi}{4} - x\right)$

$\therefore \frac{dy}{dx} = -1$.

$\because -\frac{\pi}{4} < x < \frac{\pi}{4} \Rightarrow -\frac{\pi}{2} < x - \frac{\pi}{4} < 0 \Rightarrow 0 < \frac{\pi}{4} - x < \frac{\pi}{2}$; $\cos^{-1}\cos x = x$, $x \in [0, \pi]$.

09. Absolute minimum value of $f(x) = (x-2)^2 + 5$ in the interval [–3, 2] is

(a) –3 (b) 2 (c) 5 (d) 30

Sol. (c) For $f(x) = (x-2)^2 + 5$, $f'(x) = 2(x-2)^2$.

Put $f'(x) = 0$, $2(x-2)^2 = 0 \Rightarrow x = 2$.

Now $f(-3) = (-3-2)^2 + 5 = 30$, $f(2) = (2-2)^2 + 5 = 5$.

So, the required absolute minimum value of f is 5.

10. $\int \frac{1}{\sqrt{1+\cos 2x}}dx$ is equal to

(a) $\log\cos x + C$ (b) $\frac{1}{\sqrt{2}}\log|\sec x + \tan x| + C$

(c) $\frac{1}{\sqrt{2}}\log|\sec x - \tan x| + C$ (d) $\log\sin 2x + C$

Sol. (b) $\int \frac{1}{\sqrt{1+\cos 2x}}dx = \int \frac{1}{\sqrt{2\cos^2 x}}dx$

$= \frac{1}{\sqrt{2}}\int \frac{1}{\cos x}dx$

$= \frac{1}{\sqrt{2}}\int \sec x\,dx = \frac{1}{\sqrt{2}}\log|\sec x + \tan x| + C$.

11. The value of $\int_{-5}^{-1} \frac{1}{x}dx$ is equal to

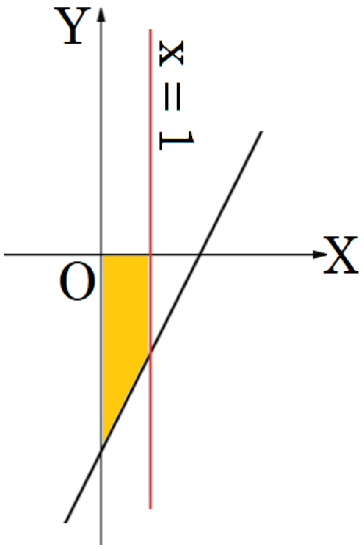
(a) $-\log 5$ (b) x^6 (c) $\log(-5)$ (d) x^{-6}

Sol. (a) $\int_{-5}^{-1} \frac{1}{x}dx = \left[\log|x|\right]_{-5}^{-1} = \log|-1| - \log|-5| = \log 1 - \log 5 = -\log 5$ [$\because \log 1 = 0$

12. An ant is observed crawling on a sheet of paper along a straight line given by equation $y = 2x - 4$. Area of the surface covered by the ant bounded by y-axis, x-axis and $x = 1$ is

(a) 1 sq. unit (b) 3 sq. units (c) 2 sq. units (d) 4 sq. units

Sol. (b) $y = 2x - 4$, y-axis, x-axis and $x = 1$.



$$\text{Required area} = \left|\int_0^1 (2x-4)\,dx\right|$$
$$= \left|\frac{1}{2\times 2}\left[(2x-4)^2\right]_0^1\right|$$
$$= \left|\frac{1}{2\times 2}[4-16]\right|$$
$$= \left|\frac{1}{2\times 2}(-12)\right|$$
$$= |-3|$$
$$= 3 \text{ Sq. units}.$$

13. The order and degree of the differential equation $1+\left(\frac{d^3y}{dx^3}\right)^3 = \lambda\frac{d^2y}{dx^2}$ is

(a) Order = 3, Degree = 3 (b) Order = 2, Degree = 2
(c) Order = 3, Degree = 1 (d) Order = 2, Degree = 1

Sol. (a) Order = 3, Degree = 3.

14. The general solution for the differential equation $\frac{dy}{dx} = e^{3x-y}$ is

(a) $3e^y = e^{3x} + C$ (b) $\log(3x-y) = C$ (c) $e^{3x-y} = C$ (d) $-e^y + 3e^{3x} = C$

Sol. (a) Rewriting the D.E., $\frac{dy}{dx} = e^{3x}\times e^{-y}$

$\Rightarrow \int e^y dy = \int e^{3x} dx$

$\Rightarrow e^y = \frac{1}{3}e^{3x} + k$

$\Rightarrow 3e^y = e^{3x} + C$, where $C = 3k$.

15. The corner points of the feasible region determined by the system of linear constraints are (0, 0), (0, 40), (20, 40), (60, 20) and (60, 0). If the objective function of an LPP is $Z = 4x + 3y$, then the maximum value is

(a) 200 (b) 300 (c) 240 (d) 120

Sol. (b) $\because Z_{(0,0)} = 0$, $Z_{(0,40)} = 120$, $Z_{(20,40)} = 200$, $Z_{(60,20)} = 300$, $Z_{(60,0)} = 240$.

$\therefore$ Maximum value of Z is 300.

16. If position vector $\vec{p}$ of a point (24, n) is such that $|\vec{p}| = 25$, then the value of n is

(a) ± 49 (b) ± 5 (c) ± 1 (d) ± 7

Sol. (d) Here $\vec{p} = 24\hat{i} + n\hat{j}$.

Since $|\vec{p}| = 25$ so, $\sqrt{24^2 + n^2} = 25$

$\Rightarrow 24^2 + n^2 = 25^2$

$\Rightarrow n^2 = 25^2 - 24^2 = (25+24)(25-24)$

$\Rightarrow n^2 = 49$

$\therefore n = \pm 7$.

17. If vectors $\vec{a} = 3\hat{i} + 2\hat{j} + \lambda\hat{k}$ and $\vec{b} = 2\hat{i} - 4\hat{j} + 5\hat{k}$ represent the two strips of the Red Cross sign placed outside a doctor's clinic, then the value of λ is

(a) 1 (b) $\frac{5}{2}$ (c) $\frac{2}{5}$ (d) 0

Sol. (c) A Red Cross consists of two strips that are **perpendicular** to each other.

So, using $\vec{a}.\vec{b}=0$ we get $(3\hat{i}+2\hat{j}+\lambda\hat{k}).(2\hat{i}-4\hat{j}+5\hat{k})=0 \Rightarrow 6-8+5\lambda=0 \quad \therefore \lambda=\frac{2}{5}$.

REMARK The question should have mentioned about the strips of Red Cross sign. Maybe giving a suitable diagram would convey the mathematical information required to solve this problem.

18. If $3P(A)=P(B)=\frac{3}{5}$ and $P(A|B)=\frac{2}{3}$, then $P(A\cup B)$ is

(a) $\frac{3}{5}$ (b) $\frac{1}{5}$ (c) $\frac{2}{15}$ (d) $\frac{2}{5}$

Sol. (d) $\because 3P(A)=P(B)=\frac{3}{5} \quad \Rightarrow P(A)=\frac{1}{5}, P(B)=\frac{3}{5}$

Also $P(A|B)=\frac{2}{3}=\frac{P(A\cap B)}{P(B)}$

$\Rightarrow \frac{2}{3}\times P(B)=P(A\cap B)$

$\Rightarrow \frac{2}{3}\times\frac{3}{5}=P(A\cap B)$

$\Rightarrow P(A\cap B)=\frac{2}{5}$

Now $P(A\cup B)=P(A)+P(B)-P(A\cap B)$

$\Rightarrow P(A\cup B)=\frac{1}{5}+\frac{3}{5}-\frac{2}{5}=\frac{2}{5}$.

REMARK Here $P(A)=0.2$, $P(A\cap B)=0.4$ i.e., $P(A\cap B)>P(A)$, which is not possible.

Direction : *Question numbers* ***19*** *and* ***20*** *are Assertion (A) and Reason (R) based questions carrying* ***1 mark*** *each. Two statements are given, one labeled Assertion (A) and other labeled Reason (R). Select the correct answer from the options given below.*

(a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).
(b) Both Assertion (A) and Reason (R) are true and Reason (R) is **not** the correct explanation of Assertion (A).
(c) Assertion (A) is true but Reason (R) is false.
(d) Assertion (A) is false but Reason (R) is true.

19. **Assertion (A) :** A relation R on the set {1, 2, 3} defined as $R=\{(1,1),(1,2),(2,1),(2,2),(3,3)\}$ is an equivalence relation.
Reason (R) : A relation that is reflexive, symmetric and transitive is an equivalence relation.

Sol. (a) As the relation R given in Assertion is reflexive, symmetric and transitive; hence it is an equivalence relation.
Both Assertion and Reason are true and, Reason is the correct explanation of Assertion.

20. **Assertion (A) :** Consider a Linear Programming Problem with minimize $Z=x+2y$ subject to constraints $2x+y\geq 3$, $x+2y\geq 6$; $x, y\geq 0$ which gives minimum Z at infinitely many points. The corner points of feasible region are (0, 3) and (6, 0).
Reason (R) : If two corner points produce the same minimum value of the objective function, then every point on the line segment joining the points will give the same minimum value.

Sol. (a) Note that, $Z_{(0,3)}=6$, $Z_{(6,0)}=6$.
So, max (Z) is obtained at all the points on the line segment joining (0, 3) and (6, 0).
$\therefore$ Both Assertion and Reason are true and, Reason is the correct explanation of Assertion.

SECTION B

This section has 5 Very Short Answer questions of 2 marks each.

21. Evaluate $\sin\left[\tan^{-1}\tan\left(\frac{3\pi}{4}\right)\right]$.

Sol. $\sin\left[\tan^{-1}\tan\left(\frac{3\pi}{4}\right)\right]=\sin\left[\tan^{-1}(-1)\right]=\sin\left[-\frac{\pi}{4}\right]=-\sin\frac{\pi}{4}=-\frac{1}{\sqrt{2}}$.

Note that, $\tan\left(\frac{3\pi}{4}\right)=\tan\left(\pi-\frac{\pi}{4}\right)=-\tan\frac{\pi}{4}=-1$.

22. (a) Differentiate x^x with respect to $x\log x$.

OR

(b) If $y=P\cos ux+Q\sin ux$, show that $\frac{d^2y}{dx^2}+u^2y=0$.

Sol. (a) Let $u=x^x$ and $v=x\log x$.

Note that $u=e^{\log x^x}=e^{x\log x}$.

Now $u=e^{x\log x}$ and $v=x\log x$

$\Rightarrow\frac{du}{dx}=e^{x\log x}\left[x\times\frac{1}{x}+\log x\times 1\right]=x^x(1+\log x)$ and $\frac{dv}{dx}=x\times\frac{1}{x}+\log x\times 1=1+\log x$

So, $\frac{du}{dv}=\frac{du}{dx}\times\frac{dx}{dv}=x^x(1+\log x)\times\frac{1}{(1+\log x)}=x^x$.

OR

(b) $y=P\cos ux+Q\sin ux$ …(i)

$\Rightarrow\frac{dy}{dx}=-uP\sin ux+uQ\cos ux$

$\Rightarrow\frac{d^2y}{dx^2}=-u^2P\cos ux-u^2Q\sin ux=-u^2(P\cos ux+Q\sin ux)$

By (i), $\frac{d^2y}{dx^2}=-u^2y$

$\therefore\ \frac{d^2y}{dx^2}+u^2y=0$.

23. Determine the values of x for which $f(x)=\frac{x-3}{x+1}$, $x\neq -1$ is an increasing function.

Sol. We have $f(x)=\frac{x-3}{x+1}$, $x\neq -1$

$\Rightarrow f(x)=1-\frac{4}{x+1}$

$\Rightarrow f'(x)=0-\left[-\frac{4}{(x+1)^2}\right]=\frac{4}{(x+1)^2}>0$ for all $x\in\mathbb{R}-\{-1\}$

Hence, f is always increasing for all $x\in\mathbb{R}-\{-1\}$.

24. (a) Three honey bees were found flying along the vectors $\vec{a}=2\hat{i}-3\hat{j}+\hat{k}$, $\vec{b}=4\hat{j}-2\hat{k}$ and $\vec{c}=3\hat{i}+2\hat{k}$ respectively. Find the value of λ such that the path for $\vec{a}+\lambda\vec{b}$ is perpendicular to $\vec{c}$.

OR

(b) If A, B and C be three non-collinear points such that $\overrightarrow{AB} = \hat{i} + 2\hat{j} - \hat{k}$ and $\overrightarrow{AC} = 2\hat{i} - 3\hat{j}$, then find the area of ΔABC.

Sol. (a) As $(\vec{a} + \lambda\vec{b}) \perp \vec{c} \quad \therefore (\vec{a} + \lambda\vec{b}).\vec{c} = 0$

Now $\left[2\hat{i} - 3\hat{j} + \hat{k} + \lambda(4\hat{j} - 2\hat{k})\right].(3\hat{i} + 2\hat{k}) = 0$

$\Rightarrow (2)(3) + (1)(2) + \lambda(-2)(2) = 0$

$\Rightarrow 6 + 2 + \lambda(-4) = 0$

$\Rightarrow \lambda(-4) = -8$

$\therefore \lambda = 2$.

OR

(b) $\overrightarrow{AB} = \hat{i} + 2\hat{j} - \hat{k}$, $\overrightarrow{AC} = 2\hat{i} - 3\hat{j}$

Now $\overrightarrow{AB} \times \overrightarrow{AC} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & -1 \\ 2 & -3 & 0 \end{vmatrix} = -3\hat{i} - 2\hat{j} - 7\hat{k}$.

So, the area of $\Delta ABC = \frac{1}{2}\left|\overrightarrow{AB} \times \overrightarrow{AC}\right| = \frac{1}{2}\sqrt{(-3)^2 + (-2)^2 + (-7)^2} = \frac{\sqrt{9+4+49}}{2} = \frac{\sqrt{62}}{2}$ Sq. units.

25. Find the angle between the following pair of lines $\frac{x-2}{3} = \frac{y+5}{2} = \frac{1-z}{-6}$ and $\frac{x-7}{1} = \frac{y}{2} = \frac{6-z}{-2}$.

Sol. Rewriting the given lines, we get $\frac{x-2}{3} = \frac{y+5}{2} = \frac{z-1}{6}$ and $\frac{x-7}{1} = \frac{y}{2} = \frac{z-6}{2}$.

Their respective direction ratios are 3, 2, 6 and 1, 2, 2.

Let θ be the required angle between both the lines.

$$\therefore \cos\theta = \frac{3\times1 + 2\times2 + 6\times2}{\sqrt{3^2 + 2^2 + 6^2}\sqrt{1^2 + 2^2 + 2^2}}$$

$$\Rightarrow \cos\theta = \frac{3+4+12}{\sqrt{9+4+36}\sqrt{1+4+4}} = \frac{19}{\sqrt{49}\sqrt{9}} = \frac{19}{7\times3}$$

$$\therefore \theta = \cos^{-1}\left(\frac{19}{21}\right).$$

SECTION C

There are 6 Short Answer questions in this section. Each is of 3 marks.

26. A spherical balloon loses its volume due to escape of air from it in such a way that decrease of volume at any instant is proportional to its surface area. Show that the radius is decreasing at a constant rate.

Sol. Given $-\frac{dV}{dt} \propto S \Rightarrow -\frac{dV}{dt} = kS$.

Now $-\frac{d}{dt}\left[\frac{4\pi}{3}\times r^3\right] = k\times4\pi r^2$

$$\Rightarrow -\left[\frac{4\pi}{3}\times3r^2\times\frac{dr}{dt}\right] = k\times4\pi r^2$$

$$\Rightarrow -\frac{dr}{dt} = k$$

$$\Rightarrow \frac{dr}{dt} = -k$$

Since k is a constant, the radius is decreasing at a constant rate.

27. (a) Find $\int \frac{x-\sin x}{1-\cos x}dx$.

OR

(b) Evaluate $\int_0^2 \frac{1}{\sqrt{x^2+2x+3}}dx$.

Sol. (a) Let $I=\int \frac{x-\sin x}{1-\cos x}dx$

$$\Rightarrow I=\int \frac{x}{1-\cos x}dx-\int \frac{\sin x}{1-\cos x}dx$$

$$\Rightarrow I=\frac{1}{2}\int x\,\text{cosec}^2\frac{x}{2}dx-\int \cot\frac{x}{2}dx$$

$$\Rightarrow I=\frac{1}{2}\int x\,\text{cosec}^2\frac{x}{2}dx-\left[\cot\frac{x}{2}\int 1dx-\int\left(\frac{d}{dx}\cot\frac{x}{2}\int 1dx\right)dx\right]$$

$$\Rightarrow I=\frac{1}{2}\int x\,\text{cosec}^2\frac{x}{2}dx-\left[x\cot\frac{x}{2}+\frac{1}{2}\int x\,\text{cosec}^2\frac{x}{2}dx\right]$$

$$\Rightarrow I=\frac{1}{2}\int x\,\text{cosec}^2\frac{x}{2}dx-x\cot\frac{x}{2}-\frac{1}{2}\int x\,\text{cosec}^2\frac{x}{2}dx$$

$$\therefore I=-x\cot\frac{x}{2}+C.$$

REMARK We can also apply integral by parts method in $\frac{1}{2}\int x\,\text{cosec}^2\frac{x}{2}dx$.

OR

(b) Let $I=\int_0^2 \frac{1}{\sqrt{x^2+2x+3}}dx$

$$\Rightarrow I=\int_0^2 \frac{1}{\sqrt{(x+1)^2+(\sqrt{2})^2}}dx$$

$$\Rightarrow I=\left[\log\left|x+1+\sqrt{(x+1)^2+(\sqrt{2})^2}\right|\right]_0^2$$

$$\Rightarrow I=\log\left|2+1+\sqrt{(2+1)^2+2}\right|-\log\left|0+1+\sqrt{(0+1)^2+2}\right|$$

$$\Rightarrow I=\log\left|3+\sqrt{11}\right|-\log\left|1+\sqrt{3}\right|$$

$$\therefore I=\log\left(\frac{3+\sqrt{11}}{1+\sqrt{3}}\right).$$

REMARK In the question paper, the given integral was $\int_0^2 \frac{1}{\sqrt{x^2+2x+3}}dx$, the square root applied only till 'x^2+2x+' and the term '3' was not inside the square root. It was a strange situation for some of the students, as they had to pre-assume that square root is on x^2+2x+3.

28. Solve the differential equation $(x+2y^3)\,dy=y\,dx$.

Sol. Rewriting the D.E., we get $\frac{x+2y^3}{y}=\frac{dx}{dy}$ $\Rightarrow \frac{dx}{dy}+\left(-\frac{1}{y}\right)x=2y^2$

On comparing with $\frac{dx}{dy}+P(y)x=Q(y)$, we get $P(y)=-\frac{1}{y}$, $Q(y)=2y^2$.

So, I.F. $=e^{\int -\frac{1}{y}dy}=e^{-\log y}=\frac{1}{e^{\log y}}=\frac{1}{y}$.

Required solution is, $\frac{1}{y}\times x=\int \frac{1}{y}\times 2y^2 dy+C$

$\Rightarrow \frac{x}{y}=\int 2y\,dy+C$

$\Rightarrow \frac{x}{y}=2\times\frac{y^2}{2}+C$

$\Rightarrow \frac{x}{y}=y^2+C$ or $x=y^3+yC$.

29. Solve the following Linear Programming Problem graphically.

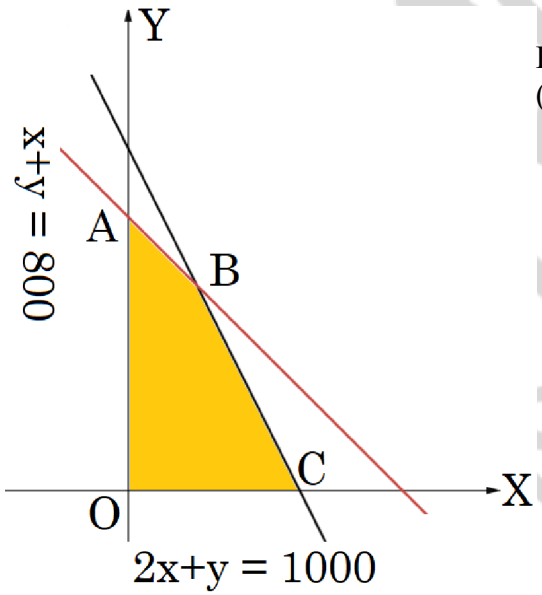
Maximise $Z=\frac{2x}{5}+\frac{3y}{10}$

Subject to the constraints $2x+y\le 1000$, $x+y\le 800$; $x, y\ge 0$.

Sol. Consider the graph shown below.

Corner points are O(0, 0), A(0, 800), B(200, 600) and C(500, 0).

$\therefore Z_O=0$, $Z_A=240$, $Z_B=260$, $Z_C=200$.



Hence, the maximum value of Z is 260 at the point (200, 600).

30. (a) Let three toys A, B and C be placed in the same straight line. If the position vectors of A, B and C are $55\hat{i}-2\hat{j}$, $5\hat{i}+8\hat{j}$ and $a\hat{i}-52\hat{j}$ respectively, find the value of 'a'.

OR

(b) If $\vec{a}$, $\vec{b}$ and $\vec{c}$ are unit vectors, then prove that $|\vec{a}-\vec{b}|^2+|\vec{b}-\vec{c}|^2+|\vec{c}-\vec{a}|^2\le 9$.

Sol. (a) $\overrightarrow{AB}=(5\hat{i}+8\hat{j})-(55\hat{i}-2\hat{j})=-50\hat{i}+10\hat{j}$, $\overrightarrow{AC}=(a\hat{i}-52\hat{j})-(55\hat{i}-2\hat{j})=(a-55)\hat{i}-50\hat{j}$

Since A, B and C are collinear so, $\overrightarrow{AB}\parallel\overrightarrow{AC}$.

Note that, for two vectors to be collinear (or parallel), their components must be proportional.

Therefore, $\frac{-50}{a-55}=\frac{10}{-50}$

$\Rightarrow \frac{-50}{a-55} = \frac{1}{-5}$

$\Rightarrow a - 55 = 250$

$\Rightarrow a = 250 + 55$

$\therefore a = 305$.

OR

(b) Let $y = |\vec{a} - \vec{b}|^2 + |\vec{b} - \vec{c}|^2 + |\vec{c} - \vec{a}|^2$.

Taking $y = |\vec{a}|^2 + |\vec{b}|^2 - 2\vec{a}.\vec{b} + |\vec{b}|^2 + |\vec{c}|^2 - 2\vec{b}.\vec{c} + |\vec{c}|^2 + |\vec{a}|^2 - 2\vec{c}.\vec{a}$

$\Rightarrow y = 2\left[|\vec{a}|^2 + |\vec{b}|^2 + |\vec{c}|^2\right] - 2(\vec{a}.\vec{b} + \vec{b}.\vec{c} + \vec{c}.\vec{a})$

$\Rightarrow y = 2\left[1^2 + 1^2 + 1^2\right] - 2(\vec{a}.\vec{b} + \vec{b}.\vec{c} + \vec{c}.\vec{a})$

$\Rightarrow y = 6 - 2(\vec{a}.\vec{b} + \vec{b}.\vec{c} + \vec{c}.\vec{a})$ ...(i) [As $\vec{a}$, $\vec{b}$ and $\vec{c}$ are unit vectors

Since the square of the magnitude of any vector is always non-negative.

So, $|\vec{a} + \vec{b} + \vec{c}|^2 \geq 0$

$\Rightarrow |\vec{a}|^2 + |\vec{b}|^2 + |\vec{c}|^2 + 2(\vec{a}.\vec{b} + \vec{b}.\vec{c} + \vec{c}.\vec{a}) \geq 0$

$\Rightarrow 1^2 + 1^2 + 1^2 + 2(\vec{a}.\vec{b} + \vec{b}.\vec{c} + \vec{c}.\vec{a}) \geq 0$

$\Rightarrow 2(\vec{a}.\vec{b} + \vec{b}.\vec{c} + \vec{c}.\vec{a}) \geq -3$

$\Rightarrow -2(\vec{a}.\vec{b} + \vec{b}.\vec{c} + \vec{c}.\vec{a}) \leq 3$

$\Rightarrow 6 - 2(\vec{a}.\vec{b} + \vec{b}.\vec{c} + \vec{c}.\vec{a}) \leq 6 + 3$

By (i), $y \leq 9$

Hence, $|\vec{a} - \vec{b}|^2 + |\vec{b} - \vec{c}|^2 + |\vec{c} - \vec{a}|^2 \leq 9$.

31. (a) A die is rolled. Consider events : $A = \{1, 2, 5\}$, $B = \{3, 5\}$, $C = \{2, 3, 4, 5\}$ and hence find
(i) $P(A \mid C)$ and $P(C \mid A)$ (ii) $P(A \cap B \mid C)$ and $P(A \cup B \mid C)$

OR

(b) A box contains 6 cards numbered 1 to 6. A student is asked to pick up two cards, one by one after replacement and note down the numbers on the cards. Let A be the event of getting sum of the numbers on two cards as 10, and B, the event of a number other than 4 on the first card selected. Find P(A and B) and find whether the events A and B are independent events or not.

Sol. Here $S = \{1, 2, 3, 4, 5, 6\}$

Given $A = \{1, 2, 5\}$, $B = \{3, 5\}$, $C = \{2, 3, 4, 5\}$.

$\therefore A \cap C = \{2, 5\}$, $A \cap B = \{5\}$, $A \cup B = \{1, 2, 3, 5\}$, $A \cap B \cap C = \{5\}$ and $A \cup B \cap C = \{2, 3, 5\}$.

(i) $P(A \mid C) = \frac{P(A \cap C)}{P(C)} = \frac{\frac{2}{6}}{\frac{4}{6}} = \frac{2}{4} = \frac{1}{2}$ and $P(C \mid A) = \frac{P(C \cap A)}{P(A)} = \frac{\frac{2}{6}}{\frac{3}{6}} = \frac{2}{3}$.

(ii) $P(A \cap B \mid C) = \frac{P(A \cap B \cap C)}{P(C)} = \frac{\frac{1}{6}}{\frac{4}{6}} = \frac{1}{4}$

and $P(A \cup B \mid C) = \dfrac{P(A \cup B \cap C)}{P(C)} = \dfrac{\frac{3}{6}}{\frac{4}{6}} = \dfrac{3}{4}$.

OR

(b) Total outcomes, $n(S) = 6^2 = 36$.

- Event A (Sum is 10): {(4, 6), (5, 5), (6, 4)}

$\therefore P(A) = \dfrac{3}{36} = \dfrac{1}{12}$.

- Event B (First card is not 4): Since there are 6 options for the first card and 5 of them are 'not 4', and 6 options for the second card. Total outcomes will be $36 - 6 = 30$.

$\therefore P(B) = \dfrac{30}{36} = \dfrac{5}{6}$.

- Event $A \cap B$ (Sum is 10 and first card is not 4): {(5, 5), (6, 4)}

$\therefore P(A \cap B) = \dfrac{2}{36} = \dfrac{1}{18}$.

Since $P(A).\ P(B) = \dfrac{1}{12} \times \dfrac{5}{6} = \dfrac{5}{72}$. Clearly, $P(A \cap B) \neq P(A).\ P(B)$.

Hence, events A and B are not independent events.

SECTION D

There are 4 Long Answer questions in this section. Each is of 5 marks.

32. A man goes to buy fruits from the market. The shopkeeper informs him that 4 apples, 3 oranges and 2 bananas cost ₹60; 2 apples, 4 oranges and 6 bananas cost ₹90; whereas 6 apples, 2 oranges and 3 bananas cost ₹70. Using matrix method, find the cost of one fruit of each kind.

Sol. Let the cost of 1 apple, 1 orange, and 1 banana be x, y and z (in ₹) respectively.

Based on the information, we have

$4x + 3y + 2z = 60$,

$2x + 4y + 6z = 90 \Rightarrow x + 2y + 3z = 45$,

$6x + 2y + 3z = 70$

Let $A = \begin{bmatrix} 4 & 3 & 2 \\ 1 & 2 & 3 \\ 6 & 2 & 3 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$, $B = \begin{bmatrix} 60 \\ 45 \\ 70 \end{bmatrix}$.

Since $AX = B \quad \Rightarrow X = A^{-1}B$ ...(i)

Now $|A| = 4(0) - 3(-15) + 2(-10) = 45 - 20 = 25 \neq 0 \quad \therefore A^{-1}$ exists.

$\because \text{adj.}A = \begin{bmatrix} 0 & -5 & 5 \\ 15 & 0 & -10 \\ -10 & 10 & 5 \end{bmatrix} \quad \therefore A^{-1} = \dfrac{\text{adj.}A}{\det.(A)} = \dfrac{1}{25}\begin{bmatrix} 0 & -5 & 5 \\ 15 & 0 & -10 \\ -10 & 10 & 5 \end{bmatrix}$

By (i), $X = \dfrac{1}{25}\begin{bmatrix} 0 & -5 & 5 \\ 15 & 0 & -10 \\ -10 & 10 & 5 \end{bmatrix}\begin{bmatrix} 60 \\ 45 \\ 70 \end{bmatrix}$

$\Rightarrow X = \dfrac{1}{25} \times 5\begin{bmatrix} 0 & -1 & 1 \\ 3 & 0 & -2 \\ -2 & 2 & 1 \end{bmatrix} \times 5\begin{bmatrix} 12 \\ 9 \\ 14 \end{bmatrix}$

$$\Rightarrow X=\begin{bmatrix}0 & -1 & 1\\ 3 & 0 & -2\\ -2 & 2 & 1\end{bmatrix}\begin{bmatrix}12\\ 9\\ 14\end{bmatrix}$$

$$\Rightarrow \begin{bmatrix}x\\ y\\ z\end{bmatrix}=\begin{bmatrix}5\\ 8\\ 8\end{bmatrix}$$

$\therefore x=5,\ y=8,\ z=8.$

Hence, the cost of one fruit of each kind is : Apple = ₹5, Orange = ₹8, Banana = ₹8.

33. (a) If $y\sqrt{x^2+1}=\log\left[\sqrt{x^2+1}-x\right]$, show that $(x^2+1)\dfrac{dy}{dx}+xy+1=0$.

OR

(b) Find the differential of $x^{\cot x}+\dfrac{2x^2-3}{2x^2-x+2}$ with respect to x.

Sol. (a) $y\sqrt{x^2+1}=\log\left[\sqrt{x^2+1}-x\right]$

$$\Rightarrow y\times\frac{1}{2\sqrt{x^2+1}}\times 2x+\sqrt{x^2+1}\times\frac{dy}{dx}=\frac{1}{\sqrt{x^2+1}-x}\times\left(\frac{1}{2\sqrt{x^2+1}}\times 2x-1\right)$$

$$\Rightarrow \frac{yx}{\sqrt{x^2+1}}+\sqrt{x^2+1}\times\frac{dy}{dx}=\frac{1}{\sqrt{x^2+1}-x}\times\left(\frac{x}{\sqrt{x^2+1}}-1\right)$$

$$\Rightarrow \frac{yx}{\sqrt{x^2+1}}+\sqrt{x^2+1}\times\frac{dy}{dx}=\frac{1}{-\left[x-\sqrt{x^2+1}\right]}\times\left(\frac{x-\sqrt{x^2+1}}{\sqrt{x^2+1}}\right)$$

$$\Rightarrow \frac{yx}{\sqrt{x^2+1}}+\sqrt{x^2+1}\times\frac{dy}{dx}=-\frac{1}{\sqrt{x^2+1}}$$

$$\Rightarrow yx+(x^2+1)\times\frac{dy}{dx}=-1$$

Hence, $(x^2+1)\dfrac{dy}{dx}+xy+1=0$.

REMARK The question paper had a typing/printing error in this Q33 (a).

It was given like this way in the paper : $y\sqrt{x^2+1}=\log\sqrt{x^2+1}-x$, the brackets in RHS were missing.

OR

(b) Let $y=x^{\cot x}+\dfrac{2x^2-3}{2x^2-x+2}$

$$\Rightarrow y=e^{\log x^{\cot x}}+\frac{(2x^2-x+2)-3+x-2}{2x^2-x+2}$$

$$\Rightarrow y=e^{\cot x\log x}+1+\frac{x-5}{2x^2-x+2}$$

$$\Rightarrow \frac{dy}{dx}=e^{\cot x\log x}\left[\cot x\times\frac{1}{x}+\log x\,(-\operatorname{cosec}^2 x)\right]+0+\frac{(2x^2-x+2)(1-0)-(x-5)(4x-1)}{(2x^2-x+2)^2}$$

$$\Rightarrow \frac{dy}{dx}=x^{\cot x}\left[\frac{\cot x}{x}-\operatorname{cosec}^2 x\log x\right]+\frac{(2x^2-x+2)-(4x^2-21x+5)}{(2x^2-x+2)^2}$$

$$\Rightarrow \frac{dy}{dx} = x^{\cot x}\left[\frac{\cot x}{x} - \text{cosec}^2 x \log x\right] + \frac{20x - 2x^2 - 3}{(2x^2 - x + 2)^2}.$$

34. Sketch the curve $\{(x, y) : 100x^2 + 25y^2 = 2500\}$ and find the area of the region enclosed by it, using integration.

Sol. We have $100x^2 + 25y^2 = 2500$ i.e., $\frac{x^2}{25} + \frac{y^2}{100} = 1$. Clearly, it is an ellipse of $\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$ form.

Also $25y^2 = 2500 - 100x^2$

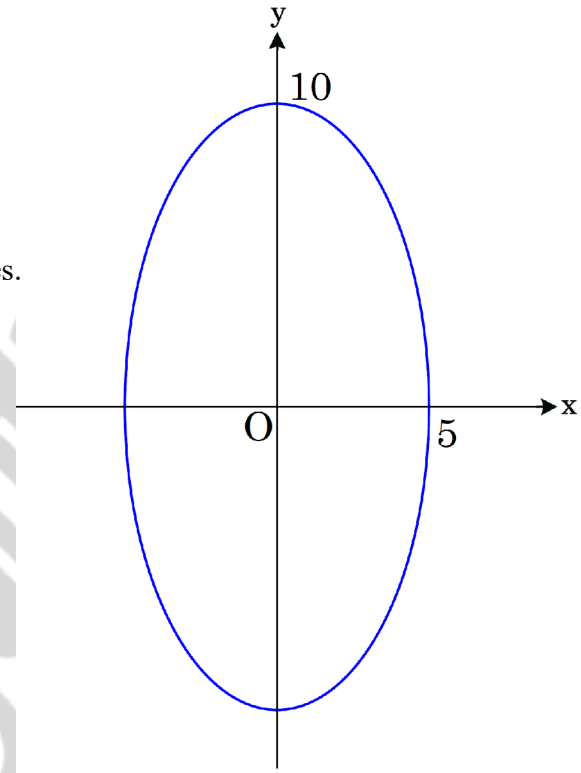
$\Rightarrow 5y = \pm\sqrt{2500 - 100x^2}$

$\Rightarrow 5y = \pm 10\sqrt{25 - x^2}$

$\Rightarrow y = \pm 2\sqrt{25 - x^2}$

Refer the diagram shown.

Also the ellipse is symmetrical about both the axes.



$$\therefore \text{Required area} = 4\int_0^5 2\sqrt{25 - x^2}\,dx$$

$$= 8\left[\frac{x}{2}\sqrt{25 - x^2} + \frac{25}{2}\sin^{-1}\frac{x}{5}\right]_0^5$$

$$= 8\left[\left(0 + \frac{25}{2}\sin^{-1}1\right) - (0 + 0)\right]$$

$$= 8\left[\left(0 + \frac{25}{2} \times \frac{\pi}{2}\right)\right]$$

$$= 50\pi \text{ Sq. units}.$$

35. (a) Find the foot of the perpendicular from the point (0, 2, 3) on the line $\frac{-x-3}{-5} = \frac{1-y}{-2} = \frac{3z+12}{9}$ and hence find the length of the perpendicular.

OR

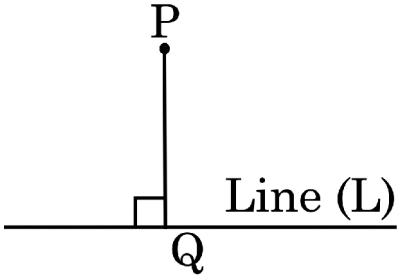
(b) Find the value of p if the shortest distance between the lines $\vec{r} = (\hat{i} + 2\hat{j} + \hat{k}) + \lambda(\hat{i} - \hat{j} + \hat{k})$ and $\vec{r} = (p\hat{i} - \hat{j} - \hat{k}) + \mu(2\hat{i} + \hat{j} + 2\hat{k})$ is $\frac{3}{\sqrt{2}}$ units.

Sol. (a) Let P(0, 2, 3).

Also let $L : \frac{-x-3}{-5} = \frac{1-y}{-2} = \frac{3z+12}{9} = \lambda$ or $\frac{x+3}{5} = \frac{y-1}{2} = \frac{z+4}{3} = \lambda$.

The d.r.'s of line L are 5, 2, 3.

Draw PQ $\perp$ line L.



The coordinates of a random point on line L :

$Q(5\lambda - 3, 2\lambda + 1, 3\lambda - 4)$.

The d.r.'s of line PQ : $5\lambda - 3 - 0, 2\lambda + 1 - 2, 3\lambda - 4 - 3$

i.e., $5\lambda - 3, 2\lambda - 1, 3\lambda - 7$.

Since $a_1a_2 + b_1b_2 + c_1c_2 = 0$ for $\perp^{er}$ lines.

So, $5(5\lambda - 3) + 2(2\lambda - 1) + 3(3\lambda - 7) = 0$

$\Rightarrow 38\lambda - 38 = 0$

$\Rightarrow \lambda = 1$

So, the Foot of perpendicular is $Q(2, 3, -1)$.

Hence, the length of the perpendicular $(PQ) = \sqrt{(2-0)^2 + (3-2)^2 + (-1-3)^2}$

$\Rightarrow (PQ) = \sqrt{4+1+16}$

$\therefore (PQ) = \sqrt{21}$ units.

OR

(b) We have $\vec{r} = (\hat{i} + 2\hat{j} + \hat{k}) + \lambda(\hat{i} - \hat{j} + \hat{k})$ and $\vec{r} = (p\hat{i} - \hat{j} - \hat{k}) + \mu(2\hat{i} + \hat{j} + 2\hat{k})$.

Here $\vec{a}_1 = (\hat{i} + 2\hat{j} + \hat{k})$, $\vec{b}_1 = (\hat{i} - \hat{j} + \hat{k})$; $\vec{a}_2 = (p\hat{i} - \hat{j} - \hat{k})$, $\vec{b}_2 = (2\hat{i} + \hat{j} + 2\hat{k})$.

$$\Rightarrow \vec{a}_2 - \vec{a}_1 = (p-1)\hat{i} - 3\hat{j} - 2\hat{k}; \ \vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -1 & 1 \\ 2 & 1 & 2 \end{vmatrix} = -3\hat{i} + 3\hat{k}$$

$$\text{As S.D.} = \frac{\left|(\vec{a}_2 - \vec{a}_1).\vec{b}_1 \times \vec{b}_2\right|}{\left|\vec{b}_1 \times \vec{b}_2\right|} = \frac{3}{\sqrt{2}} \text{ units}$$

$$\Rightarrow \frac{\left|((p-1)\hat{i} - 3\hat{j} - 2\hat{k}).(-3\hat{i} + 3\hat{k})\right|}{\sqrt{(-3)^2 + 0^2 + 3^2}} = \frac{3}{\sqrt{2}}$$

$$\Rightarrow \frac{|(-3p+3) - 6|}{\sqrt{9+9}} = \frac{3}{\sqrt{2}}$$

$$\Rightarrow \frac{|(-3p-3)|}{3\sqrt{2}} = \frac{3}{\sqrt{2}}$$

$\Rightarrow |(-3p-3)| = 9$

$\Rightarrow (-3p-3) = 9$ or $(-3p-3) = -9$

$\Rightarrow (-3p) = 12$ or $(-3p) = -6$

$\therefore p = -4$ or $p = 2$.

Hence, the values of p are 2 or –4.

SECTION E

In this section there are 3 Case Study questions of 4 marks each.

36. A school wants the students of class XII to do a project on 'Sustainability' keeping the world environment in mind. They select the student participants on the basis of an essay writing competition.

7 students out of 80 are selected for the project and are categorized into two sets such that :

Girl students belong to Set $A = \{G_1, G_2, G_3, G_4\}$,

Boy students belong to Set $B = \{B_1, B_2, B_3\}$.

Based on the above information, answer the following questions.

(i) How many relations are possible from Set A $\rightarrow$ Set B?

(ii) Let R be a relation from $A \rightarrow B$ such that $R = \{(G_1, B_1), (G_2, B_2), (G_3, B_2), (G_4, B_3), (G_1, B_2)\}$. Is R an injective function? Justify your answer.

(iii) (a) Let the relation R from $A \rightarrow A$ be such that $R = \{(x, y) : x, y \in A, x \text{ and } y \text{ are students from the same colony in the city}\}$. Verify if R is an equivalence relation.

OR

(iii) (b) Verify if any function $f : B \rightarrow A$ is bijective. Give reason to support your answer.

Sol. **(i)** $\because n(A) = 4,\ n(B) = 3$.

$\therefore$ Number of relations possible from Set A to Set B $= 2^{4\times3} = 2^{12} = 4096$.

(ii) No, R is not an injective function because it is **not even a function** (due to $G_1 \in A$ having multiple images). Also, it fails the injectivity test as multiple inputs G_2, G_3 map to the same output B_2.

(iii) (a) $R = \{(x, y) : x,\ y \in A$, x and y are students from the same colony in the city$\}$.

For R to be an equivalence relation, R must be Reflexive, Symmetric, and Transitive.

- Reflexive: For any student $x \in A$, $(x, x) \in R$ because every student lives in the same colony as themselves. So, R is reflexive.
- Symmetric: If $(x, y) \in R$, then x and y are in the same colony. This implies, y and x are in the same colony, so $(y, x) \in R$. Thus, R is symmetric.
- Transitive: If $(x, y) \in R$ and $(y, z) \in R$, then x and y are in the same colony, and y and z are in the same colony. Therefore, x and z must be in the same colony, so $(x, z) \in R$. Thus, R is transitive.

Since R is reflexive, symmetric and transitive, it is an equivalence relation.

OR

(iii) (b) A function is **bijective** if it is both **injective** (one-to-one) and **surjective** (onto).

For a function to be bijective, the number of elements in the domain and codomain must be equal i.e., $n(B) = n(A)$. Note that, $n(A) = 4,\ n(B) = 3$.

Since $n(B) < n(A)$, it is impossible for the function to be onto. There will always be at least one student in Set A who is not mapped to by an element in Set B.

Therefore, the function cannot be bijective.

37. There are three types of vaccines A_1, A_2, A_3, available in the market to protect the population of the country from spread of certain infection. According to a survey conducted, it was found that 25% of the population was given Vaccine A_1, 35% of the population was given Vaccine A_2 and 40% of the population was given Vaccine A_3. The survey also stated that the probabilities that Vaccines A_1, A_2 and A_3 would protect against the infection were 60%, 55% and 50% respectively.

Based on the above information, answer the following questions.

Find the probability that

(i) the person taking vaccine A_2 will get infected.

(ii) if a person is chosen randomly, he/she will be protected from the infection.

(iii) (a) the person was given Vaccine A_1, given that the randomly chosen person is infected.

OR

(iii) (b) the person was given Vaccine A_3, given that the randomly chosen person is not infected.

Sol. Given $P(A_1) = 25\%,\ P(A_2) = 35\%,\ P(A_3) = 40\%$.

Let E be the event that a person is protected and F be the event that a person is infected.

So, $P(E \mid A_1) = 60\%,\ P(E \mid A_2) = 55\%,\ P(E \mid A_3) = 50\%$.

(i) A person taking A_2 is either protected or infected. Since the protection rate is 55%.

So, $P(F \mid A_2) = 1 - P(E \mid A_2) = 1 - 55\% = 1 - \frac{55}{100} = \frac{45}{100} = 45\% = 0.45 = \frac{9}{20}$.

(ii) Using total probability, $P(E) = P(A_1)P(E \mid A_1) + P(A_2)P(E \mid A_2) + P(A_3)P(E \mid A_3)$

$$\Rightarrow P(E)=\frac{25}{100}\times\frac{60}{100}+\frac{35}{100}\times\frac{55}{100}+\frac{40}{100}\times\frac{50}{100}$$

$$\Rightarrow P(E)=\frac{1500}{10000}+\frac{1925}{10000}+\frac{2000}{10000}=\frac{5425}{10000}=54.25\%\text{ or }0.5425\text{ or }\frac{217}{400}.$$

(iii) (a) Note that $P(F)=1-P(E)=1-0.5425=0.4575$.

Using Bayes' theorem, $P(A_1\,|\,F)=\dfrac{P(A_1)P(F\,|\,A_1)}{P(F)}$

$$\Rightarrow P(A_1\,|\,F)=\frac{0.25\times[1-P(E\,|\,A_1)]}{0.4575}$$

$$\Rightarrow P(A_1\,|\,F)=\frac{0.25\times[1-0.60]}{0.4575}$$

$$\Rightarrow P(A_1\,|\,F)=\frac{0.25\times0.40}{0.4575}$$

$$\Rightarrow P(A_1\,|\,F)=\frac{0.25\times0.4}{0.4575}$$

$$\Rightarrow P(A_1\,|\,F)=\frac{100}{457.5}=\frac{1000}{4575}=\frac{40}{183}.$$

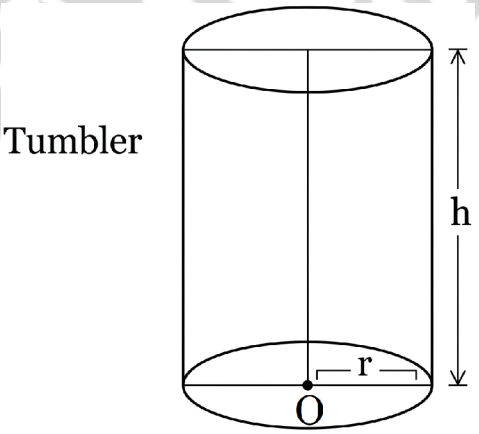
OR

(iii) (b) Using Bayes' theorem, $P(A_3\,|\,E)=\dfrac{P(A_3)P(E\,|\,A_3)}{P(E)}$

$$\Rightarrow P(A_3\,|\,E)=\frac{0.40\times0.50}{0.5425}$$

$$\Rightarrow P(A_3\,|\,E)=\frac{20}{54.25}=\frac{2000}{5425}=\frac{80}{217}.$$

38. A company produce cylindrical tumblers, open from the top. Since they want uniformity in the product, they fix the surface area of the tumblers produced.



Based on the above information, answer the following questions.

If for a tumbler, V is its volume, h the height and r the radius of the circular base, then

(i) differentiate its volume with respect to radius of the base, where the surface area is constant.

(ii) if the company wants to maximize the volume of each tumbler, then establish a relation between its height and the radius of the base.

Sol. **(i)** Surface area of cylindrical tumbler, $S=2\pi rh+\pi r^2$

$$\Rightarrow h=\frac{S-\pi r^2}{2\pi r}.$$

Also the volume, $V = \pi r^2 h = \pi r^2 \times \frac{S - \pi r^2}{2\pi r} = \frac{1}{2}(Sr - \pi r^3)$

Now $\frac{dV}{dr} = \frac{1}{2}(S - 3\pi r^2)$.

(ii) Put $\frac{dV}{dr} = 0$, $\frac{1}{2}(S - 3\pi r^2) = 0 \quad \Rightarrow (S - 3\pi r^2) = 0 \quad \Rightarrow 3\pi r^2 = S$

$\Rightarrow r = \sqrt{\frac{S}{3\pi}}$

Now $\frac{d^2V}{dr^2} = \frac{1}{2}(0 - 3\pi \times 2r) = -3\pi r < 0$ at $r = \sqrt{\frac{S}{3\pi}}$.

Hence, volume (V) is maximum.

As $S = 2\pi r h + \pi r^2$

$\Rightarrow 3\pi r^2 = 2\pi r h + \pi r^2$

$\Rightarrow 2\pi r^2 = 2\pi r h$

$\Rightarrow r = h$

To maximize the volume of the tumbler for a fixed surface area, the height must be equal to the radius of the base.

☑ Q.P. Code - 65/5/2

01. If $A = \begin{bmatrix} \sin\theta & \cos\theta \\ -\cos\theta & \sin\theta \end{bmatrix}$ and $A + A' = I$, then θ is equal to

(a) 0 (b) $\frac{\pi}{6}$ (c) $\frac{\pi}{3}$ (d) π

Sol. (b) Since $A + A' = I \Rightarrow \begin{bmatrix} \sin\theta & \cos\theta \\ -\cos\theta & \sin\theta \end{bmatrix} + \begin{bmatrix} \sin\theta & -\cos\theta \\ \cos\theta & \sin\theta \end{bmatrix} = I$

$\Rightarrow \begin{bmatrix} 2\sin\theta & 0 \\ 0 & 2\sin\theta \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

On comparing the corresponding elements, we get $2\sin\theta = 1$

$\Rightarrow \sin\theta = \frac{1}{2}$

$\therefore \theta = \frac{\pi}{6}$.

02. If A is a symmetric matrix, then for any matrix B of order same as A, BAB' is a/an

(a) Skew symmetric matrix (b) Identity matrix

(c) Symmetric matrix (d) Null matrix

Sol. (c) Let $P = BAB'$

$\Rightarrow P' = (BAB')' = BA'B' = BAB' = P \qquad [\because A = A'$

Hence, $P = BAB'$ is a symmetric matrix.

10. $\int \sqrt{\frac{1 - \cos 2x}{1 + \cos 2x}}\, dx$ is equal to

(a) $\log|\sec^2 x| + C$ (b) $\log|\cos x| + C$ (c) $\log|1 + \cos 2x| + C$ (d) $\log|\sec x| + C$

Sol. (d) $\int \sqrt{\frac{1 - \cos 2x}{1 + \cos 2x}}\, dx = \int \sqrt{\frac{2\sin^2 x}{2\cos^2 x}}\, dx = \int \sqrt{\tan^2 x}\, dx = \int \tan x\, dx = \log|\sec x| + C$.

11. The value of $\int_0^1 \frac{1}{3x-4}dx$ is

(a) $\frac{1}{3}\log 4$ (b) $-\frac{1}{3}\log 4$ (c) $\log(-4)$ (d) $\log 4$

Sol. (b) $\int_0^1 \frac{1}{3x-4}dx = \frac{1}{3}\left[\log|3x-4|\right]_0^1 = \frac{1}{3}\left[\log|-1|-\log|-4|\right] = -\frac{1}{3}\log 4$ $[\because \log|-1| = \log 1 = 0$

18. The order and degree of differential equation $\frac{d^2y}{dx^2} = 1-\left(\frac{d^3y}{dx^3}\right)^2$ is

(a) Order = 3, Degree = 3 (b) Order = 2, Degree = 2

(c) Order = 3, Degree = 2 (d) Order = 2, Degree = 1

Sol. (c) Order = 3, Degree = 2.

21. Evaluate $\sin\left[\cos^{-1}\cos\left(\frac{7\pi}{6}\right)\right]$.

Sol. $\sin\left[\cos^{-1}\cos\left(\frac{7\pi}{6}\right)\right] = \sin\left[\cos^{-1}\cos\left(\pi+\frac{\pi}{6}\right)\right] = \sin\left[\cos^{-1}\left(-\cos\frac{\pi}{6}\right)\right]$

$= \sin\left[\pi-\cos^{-1}\left(\cos\frac{\pi}{6}\right)\right]$ $[\because \cos^{-1}(-x) = \pi-\cos^{-1}x$

$= \sin\left[\pi-\frac{\pi}{6}\right] = \sin\frac{\pi}{6} = \frac{1}{2}$.

23. Determine the interval (s) in which $f(x) = 5x^{3/2}-3x^{5/2}$, $x>0$ is increasing.

Sol. For $f(x) = 5x^{3/2}-3x^{5/2}$, $x>0$

$\Rightarrow f'(x) = \frac{15}{2}x^{1/2}-\frac{15}{2}x^{3/2} = \frac{15}{2}x^{1/2}\left[1-x\right]$

Put $f'(x)=0$, $\frac{15}{2}x^{1/2}\left[1-x\right]=0$ $\therefore x=1$ $[\because x>0$

Note that, $f'(x)\geq 0$ for $x\in(0,1]$.

Hence, f is increasing in the interval $x\in(0,1]$.

26. A thin metallic wire in the shape of a circular ring has its enclosed area increasing at a uniform rate when heated. Show that the rate of change of circumference varies inversely as the radius.

Sol. The area is increasing at a uniform (constant) rate. So, $\frac{dA}{dt}=k$ (where k is a constant).

$\Rightarrow \frac{d}{dt}\left[\pi r^2\right]=k$

$\Rightarrow 2\pi r\frac{dr}{dt}=k$

$\Rightarrow \frac{dr}{dt}=\frac{k}{2\pi r}$ ...(i)

Now taking circumference as $C=2\pi r$

$\Rightarrow \frac{dC}{dt}=2\pi\frac{dr}{dt}=2\pi\times\frac{k}{2\pi r}=\frac{k}{r}$

$\Rightarrow \frac{dC}{dt}\propto\frac{1}{r}$.

Hence, the rate of change of circumference varies inversely as the radius.

28. Solve the differential equation $y\,dx + (x - y^3)\,dy = 0$.

Sol. Rewriting the D.E., we get $\frac{dx}{dy} + \frac{1}{y}x = y^2$.

On comparing with $\frac{dx}{dy} + P(y)x = Q(y)$, we get $P(y) = \frac{1}{y}$, $Q(y) = y^2$.

So, I.F. $= e^{\int \frac{1}{y} dy} = e^{\log y} = y$.

Required solution is given by $y \times x = \int y \times y^2 dy + C$

$\Rightarrow yx = \int y^3 dy + C$

$\Rightarrow yx = \frac{1}{4}y^4 + C$ or $x = \frac{y^3}{4} + \frac{C}{y}$.

32. Find cost (per kg) of each fertilizer A, B and C that the farmer needs to buy such that 1 kg each of fertilizer A and C added to 2 kg of B costs him ₹400. Also, cost of each kg of fertilizer B and C added together is equal to cost of 1 kg of fertilizer A. However, cost of 3 kg of fertilizer B added to ₹200 is the same as cost of 1 kg of fertilizer A and C together. Use matrix method to find the solution.

Sol. Let the cost per kg of fertilizer A, B and C be x, y and z respectively (in ₹).

Then according to the question, we have the following equations.

$$x + 2y + z = 400$$

$$y + z = x \Rightarrow -x + y + z = 0$$

$$3y + 200 = x + z \Rightarrow -x + 3y - z = -200$$

Let $A = \begin{bmatrix} 1 & 2 & 1 \\ -1 & 1 & 1 \\ -1 & 3 & -1 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$, $B = \begin{bmatrix} 400 \\ 0 \\ -200 \end{bmatrix}$.

Since $AX = B \quad \Rightarrow X = A^{-1}B$ …(i)

Now $|A| = 1(-4) - 2(2) + 1(-2) = -10 \neq 0 \quad \therefore A^{-1}$ exists.

$\because \text{adj.}A = \begin{bmatrix} -4 & 5 & 1 \\ -2 & 0 & -2 \\ -2 & -5 & 3 \end{bmatrix} \quad \therefore A^{-1} = \frac{\text{adj.}A}{|A|} = \frac{1}{-10}\begin{bmatrix} -4 & 5 & 1 \\ -2 & 0 & -2 \\ -2 & -5 & 3 \end{bmatrix}$

By (i), $X = \frac{1}{-10}\begin{bmatrix} -4 & 5 & 1 \\ -2 & 0 & -2 \\ -2 & -5 & 3 \end{bmatrix}\begin{bmatrix} 400 \\ 0 \\ -200 \end{bmatrix}$

$\Rightarrow X = \frac{1}{-10}\begin{bmatrix} -4 & 5 & 1 \\ -2 & 0 & -2 \\ -2 & -5 & 3 \end{bmatrix}\begin{bmatrix} 400 \\ 0 \\ -200 \end{bmatrix} = \frac{1}{-10}\begin{bmatrix} -1800 \\ -400 \\ -1400 \end{bmatrix}$

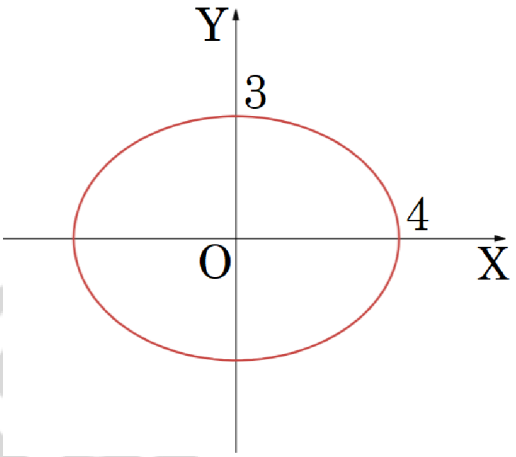
$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 180 \\ 40 \\ 140 \end{bmatrix}$

$\therefore x = 180,\ y = 40,\ z = 140$.

Hence, the costs per kg of the fertilizers A, B and C are respectively ₹180, ₹40 and ₹140.

35. Sketch the curve described by $\{(x,y): 9x^2+16y^2=144\}$ and find the area of the region enclosed by it, using integration.

Sol. We have $9x^2+16y^2=144 \Rightarrow \frac{x^2}{16}+\frac{y^2}{9}=1$, which is an ellipse of the type $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$.



Now $9x^2+16y^2=144$

$\Rightarrow 16y^2=144-9x^2$

$\Rightarrow 4y=\pm\sqrt{144-9x^2}$

$\Rightarrow y=\pm\frac{3}{4}\sqrt{16-x^2}$

Since the ellipse is symmetric across both the axes.

Hence, required area $=4\int_0^4 \frac{3}{4}\sqrt{16-x^2}\,dx$

$=3\left[\frac{x}{2}\sqrt{16-x^2}+\frac{16}{2}\sin^{-1}\frac{x}{4}\right]_0^4$

$=3\left[\left(0+8\sin^{-1}1\right)-\left(0+8\sin^{-1}0\right)\right]$

$=3\left[8\times\frac{\pi}{2}-0\right]$

$=12\pi$ Sq. units.

☑ Q.P. Code - 65/5/3

01. For any square matrix A with real entries, if $A+A'$ is a symmetric matrix, then

(a) $(A-A')$ cannot be a skew symmetric matrix

(b) $(A-A')$ is a skew symmetric matrix

(c) A is always a symmetric matrix

(d) A is always a skew symmetric matrix

Sol. (b) $(A-A')$ is a skew symmetric matrix, for any square matrix A with real entries.

02. A matrix $B=[b_{ij}]_{m\times m}$ is said to be a diagonal matrix, if

(a) $b_{ij}=0$, when $i=j$ (b) $b_{ij}=1$, when $i=j$

(c) $b_{ij}=1$, when $i\neq j$ (d) $b_{ij}=0$, when $i\neq j$

Sol. (d) By definition, a diagonal matrix is a square matrix in which all the elements outside the main diagonal are zero (0). Also for non-diagonal elements, $i\neq j$.

10. $\int\sqrt{\frac{1+\cos x}{1-\cos x}}\,dx$ is equal to

(a) $2\log\left|\sin\frac{x}{2}\right|+C$ (b) $\frac{1}{2}\log|\sin 2x|+C$ (c) $\log|1-\cos 2x|+C$ (d) $\log|1+\cos 2x|+C$

Sol. (a) $\int\sqrt{\frac{1+\cos x}{1-\cos x}}\,dx=\int\sqrt{\frac{2\cos^2\frac{x}{2}}{2\sin^2\frac{x}{2}}}\,dx=\int\sqrt{\cot^2\frac{x}{2}}\,dx=\int\cot\frac{x}{2}\,dx=2\log\left|\sin\frac{x}{2}\right|+C$.

11. If $\int_0^1(6x^2-4x+k)dx=0$, then the value of k is

(a) 1 (b) 0 (c) 2 (d) –1

Sol. (b) $\int_0^1 (6x^2 - 4x + k)dx = 0$

$$\Rightarrow \left[6 \times \frac{x^3}{3} - 4 \times \frac{x^2}{2} + k\,x\right]_0^1 = 0$$

$$\Rightarrow \left[2x^3 - 2x^2 + k\,x\right]_0^1 = 0$$

$$\Rightarrow \left[2 \times 1^3 - 2 \times 1^2 + k \times 1\right] - \left[2 \times 0^3 - 2 \times 0^2 + k \times 0\right] = 0$$

$$\Rightarrow [2 - 2 + k] - 0 = 0$$

$\therefore k = 0$.

13. The order and degree of differential equation $y = \left(\frac{d^2y}{dx^2}\right)^2 - \lambda \frac{dy}{dx}$ is

(a) Order = 2, Degree = 2 (b) Order = 2, Degree = 3

(c) Order = 1, Degree = 2 (d) Order = 2, Degree = 4

Sol. (a) Order = 2, Degree = 2.

21. Evaluate $\tan\left[\cos^{-1}\left(\tan\frac{3\pi}{4}\right)\right]$.

Sol. $\tan\left[\cos^{-1}\left(\tan\frac{3\pi}{4}\right)\right] = \tan\left[\cos^{-1}\left(\tan\left(\pi - \frac{\pi}{4}\right)\right)\right]$

$$= \tan\left[\cos^{-1}\left(-\tan\frac{\pi}{4}\right)\right]$$

$$= \tan\left[\cos^{-1}(-1)\right] \qquad [\because \cos^{-1}(-x) = \pi - \cos^{-1} x$$

$$= \tan\left[\pi - \cos^{-1}(1)\right] = \tan[\pi - 0] = \tan\pi = 0.$$

23. Determine the interval (s) in which $f(x) = \frac{x}{3} + \frac{3}{x}$, $x \neq 0$ is increasing.

Sol. Given $f(x) = \frac{x}{3} + \frac{3}{x}$, $x \neq 0$

$$\Rightarrow f'(x) = \frac{1}{3} - \frac{3}{x^2}$$

For critical points, $f'(x) = 0$

$$\Rightarrow \frac{1}{3} - \frac{3}{x^2} = 0$$

$$\Rightarrow x^2 = 9$$

$\therefore x = \pm 3$

Note that $f(x)$ is not defined at $x = 0$.

Interval	Sign of $f'(x)$	$f(x)$ is
$(-\infty, -3]$	Positive	Increasing
$[-3, 0)$	Negative	Decreasing
$(0, 3]$	Negative	Decreasing
$[3, \infty)$	Positive	Increasing

$\therefore f(x)$ is increasing on $(-\infty,-3]\cup[3,\infty)$.

26. The volume of a wooden block in the shape of a cube increases at a constant rate as the air becomes moist during the rainy season. Show that the rate of change of its surface area varies inversely as the length of edge of the cube.

Sol. Given $\frac{dV}{dt}=k$, where k is a constant.

$\Rightarrow \frac{d(a^3)}{dt}=k$, where 'a' is the edge length of cube

$\Rightarrow 3a^2\times\frac{da}{dt}=k$

$\Rightarrow \frac{da}{dt}=\frac{k}{3a^2}$ ...(i)

Now surface area of cube, $S=6a^2$

$\Rightarrow \frac{dS}{dt}=12a\times\frac{da}{dt}$

By (i), $\frac{dS}{dt}=12a\times\frac{k}{3a^2}=\frac{4k}{a}$

$\therefore \frac{dS}{dt}\propto\frac{1}{a}$.

This proves that the rate of change of the surface area varies inversely as the length of the edge of the cube.

28. Solve the differential equation $(x-\sin y)dy+\tan y\,dx=0$.

Sol. Rewriting the D.E., $\frac{dx}{dy}+\frac{x}{\tan y}=\frac{\sin y}{\tan y}$ $\Rightarrow \frac{dx}{dy}+(\cot y)x=\cos y$

On comparing with $\frac{dx}{dy}+P(y)x=Q(y)$, we get $P(y)=\cot y,\ Q(y)=\cos y$.

Now I.F. $=e^{\int \cot y dy}=e^{\log\sin y}=\sin y$.

Therefore, the required solution is $\sin y\times x=\int \sin y\cos y\,dy+C$

$\Rightarrow x\sin y=\frac{1}{2}\sin^2 y+C$ $\left[\because \int[f(x)]^n\times f'(x)dx=\frac{[f(x)]^{n+1}}{n+1}+C\right.$

$\therefore x=\frac{1}{2}\sin y+C\,\text{cosec}\,y$

A few other possible solutions are:

- Since $x\sin y=\int \sin y\cos y\,dy+C$ $\Rightarrow x\sin y=-\int(-\sin y)\cos y\,dy+C$

$\Rightarrow x\sin y=-\frac{1}{2}\cos^2 y+C$.

- Since $x\sin y=\int \sin y\cos y\,dy+C$ $\Rightarrow x\sin y=\frac{1}{2}\int \sin 2y\,dy+C$

$\Rightarrow x\sin y=\frac{1}{2}\left[-\frac{1}{2}\cos 2y\right]+C$

$\Rightarrow x\sin y=-\frac{1}{4}\cos 2y+C$.

32. Three students A, B and C go to a book-store to buy art books, story books and puzzle solving books. A buys one of each type of book for a total of ₹21. B buys 4 art books, 3 story books and 2 puzzle solving books for ₹60. C buys 6 art books, 2 story books and 3 puzzle solving books and pays ₹10 more than B. Use matrix method to find the cost of each type of book.

Sol. Let the cost (in ₹) of one art book be x, one story book be y, and one puzzle-solving book be z. Then according to the question, we get the following equations.

$$x + y + z = 21,$$
$$4x + 3y + 2z = 60,$$
$$6x + 2y + 3z = 70$$

Let $A = \begin{bmatrix} 1 & 1 & 1 \\ 4 & 3 & 2 \\ 6 & 2 & 3 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, B = \begin{bmatrix} 21 \\ 60 \\ 70 \end{bmatrix}$.

Clearly, $AX = B$

$\Rightarrow X = A^{-1}B$ …(i)

Now $|A| = 1(5) - 1(0) + 1(8 - 18) = -5 \neq 0$

$\therefore A^{-1}$ exists.

$\because \text{adj.}A = \begin{bmatrix} 5 & -1 & -1 \\ 0 & -3 & 2 \\ -10 & 4 & -1 \end{bmatrix}$ $\quad \therefore A^{-1} = \frac{1}{-5}\begin{bmatrix} 5 & -1 & -1 \\ 0 & -3 & 2 \\ -10 & 4 & -1 \end{bmatrix}$

By (i), $X = \frac{1}{-5}\begin{bmatrix} 5 & -1 & -1 \\ 0 & -3 & 2 \\ -10 & 4 & -1 \end{bmatrix}\begin{bmatrix} 21 \\ 60 \\ 70 \end{bmatrix}$

$$X = \frac{1}{-5}\begin{bmatrix} 5 & -1 & -1 \\ 0 & -3 & 2 \\ -10 & 4 & -1 \end{bmatrix}\begin{bmatrix} 21 \\ 60 \\ 70 \end{bmatrix}$$

$$\Rightarrow X = \frac{1}{-5}\begin{bmatrix} 105 - 60 - 70 \\ 0 - 180 + 140 \\ -210 + 240 - 70 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 5 \\ 8 \\ 8 \end{bmatrix}$$

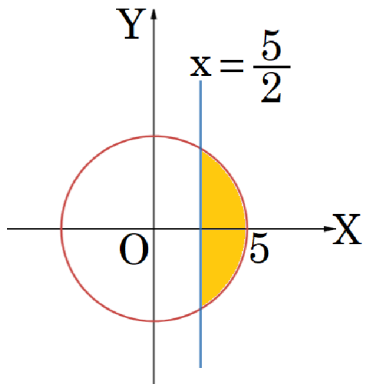
$\therefore x = 5, y = 8, z = 8$.

Hence, the cost of an Art book: ₹5, Story book: ₹8, Puzzle solving book: ₹8.

34. Sketch the graph defined by $\left\{(x, y) : \frac{x^2}{25} + \frac{y^2}{25} = 1\right\}$. Find the area of the region of minor segment cut off by the line $x = \frac{5}{2}$, using integration.

Sol. Given $\frac{x^2}{25} + \frac{y^2}{25} = 1$

$\Rightarrow x^2 + y^2 = 25$, which is a circle of radius 5 units and centre at (0, 0).



Now $x^2+y^2=25$

$\Rightarrow y^2=25-x^2$

$\Rightarrow y=\pm\sqrt{25-x^2}$

Because of the symmetry of the circle about the x-axis, the total required area is twice the area of the region in first quadrant.

$\therefore$ Required area $=2\int_{\frac{5}{2}}^{5}\sqrt{25-x^2}\,dx$

$$\Rightarrow \text{Required area}=2\left[\frac{x}{2}\sqrt{25-x^2}+\frac{25}{2}\sin^{-1}\frac{x}{5}\right]_{\frac{5}{2}}^{5}$$

$$=\left[x\sqrt{25-x^2}+25\sin^{-1}\frac{x}{5}\right]_{\frac{5}{2}}^{5}$$

$$=\left[0+25\sin^{-1}1\right]-\left[\frac{5}{2}\sqrt{25-\frac{25}{4}}+25\sin^{-1}\frac{1}{2}\right]$$

$$=25\times\frac{\pi}{2}-\left[\frac{5\times5}{2}\times\frac{\sqrt{3}}{2}+25\times\frac{\pi}{6}\right]$$

$$=25\times\frac{\pi}{3}-\frac{25\sqrt{3}}{4}$$

$$=25\left[\frac{\pi}{3}-\frac{\sqrt{3}}{4}\right]\text{ Sq. units or }\frac{25}{12}\left[4\pi-3\sqrt{3}\right]\text{ Sq. units.}$$

With regards,
O.P. GUPTA
Math Mentor
Author - MATHMISSION Series
Indira Award Winner

The O.P. Gupta Advanced Math Classes
@ **Think Academy**, Najafgarh

Telegram / WhatsApp : +919650350480

YouTube.com/@theopgupta

The Series PQSR5 ♦ Q.P. Code 65/5/1; 65/5/2; 65/5/3 is of Chennai, Bhubaneswar, Raipur and Gurugram region.

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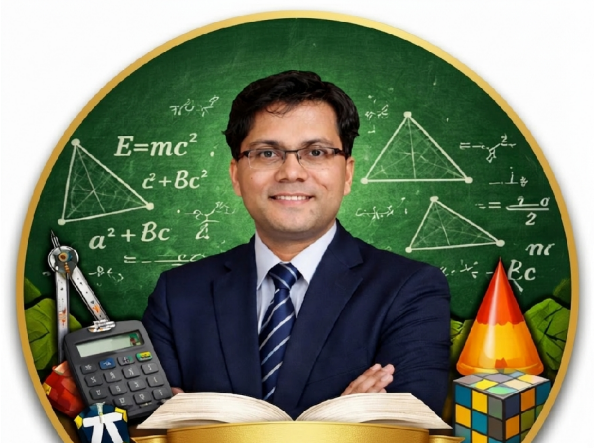
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O.P. Gupta
Author - Mathmission Series of Books
Founder & Mentor
THE O.P. GUPTA ADVANCED MATH CLASSES
@ Thana Road, Najafgarh, New Delhi

WhatsApp: +91 9650350480



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